PREPRINT · NOT PEER-REVIEWED

gr-qc · hep-th

v1 · openxiv:gr-qc.2026.00002

Second-law check through the inner Cauchy horizon of regular black holes with nonlocal fakeon-regulated mass inflation

David Alfyorov · Igor Shnyukov ·

Corresponding author: davidich.alfyorov@gmail.com; Independent researchers

CITE AS
openxiv:gr-qc.2026.00002

POSTED
2026-05-21

ISSN
3120-9556 (online)

VERSION
v1

SUBJECT
gr-qc (+1 cross-listed)

AI DISCLOSURE
COAUTHOR

COVER EVIDENCE
TRANSPARENCY · partial
IDENTITY · strong
PROVENANCE · strong
CITATIONS · strong
MATH · partial
INTEGRITY · partial

Second-law check through the inner Cauchy horizon of regular black holes with nonlocal fakeon-regulated mass inflation

David Alfyorov* Igor Shnyukov

Independent researchers

May 21, 2026

Abstract

The classical inner Cauchy horizon of a regular black hole develops exponential mass inflation under generic perturbations. The recently proposed fakeon Pauli–Villars regulator from the spectral causal theory (SCT) — a one-loop effective gravitational theory with entire-function nonlocal form factors F_1, F_2 acting on the Weyl- and Ricci-squared operators — truncates this divergence at the level of the effective Ori–Poisson–Israel equation. Two regularisation modes have been analysed: the algebraic Anselmi-smooth interpolation ("S3") and the first-principles principal-value prescription ("S4"). Working in the S3 regime, where the regulated mass aspect $\delta m_{\mathrm{MS}}(v)$ grows monotonically and saturates, we study whether the canonical entropy current of the spectrally fixed Hayward+SCT background grows monotonically along the inner-horizon evolution. Given the working ansatz that the canonical entropy is the sum of Bekenstein–Hawking area terms applied to both horizons (including the dynamically perturbed inner horizon) plus the spectral-action logarithmic correction at each, that the higher-derivative nonlocal entropy correction S_{NL} is suppressed by $(\Lambda/M_{\mathrm{Pl}})^2$, and that the Hayward functional form is the leading-curvature SCT vacuum ansatz, the chain rule yields opposite signs for the two contributions ($\mathrm{d}A_+/\mathrm{d}v > 0$ and $\mathrm{d}A_-/\mathrm{d}v < 0$), so the second law is not automatic but reduces to a non-trivial inequality between two opposite-sign terms. An asymptotic argument shows the inner-horizon contribution is suppressed as $\mathcal{O}(1/M^2)$ in the Schwarzschild-like limit (a sharper suppression than naive expectation, traceable to the spectral lock $r_-^2 - 1/(\Lambda^2 z_1) \sim 1/(2M)$ near the asymptote), and a 60-cell production sweep spanning $M\Lambda \in [0.84,\ 10^6]$ (including five near-extremal cells just above the spectrally fixed subextremal bound $M\Lambda = \sqrt{27/(16z_1)} \simeq 0.83595$) and five Price-tail exponents verifies $\mathrm{d}S_{\mathrm{total}}/\mathrm{d}v \geq 0$ at machine precision in every cell. The S4 principal-value regime, in which $\delta m_{\mathrm{MS}}(v)$ acquires a sign-flip past the crossover advanced time, requires a separate analysis of the indefinite-metric fakeon flux and is not covered here.

1 Introduction

The classical inner Cauchy horizon of a charged or rotating black hole is generically unstable: ingoing and outgoing late-time fluxes are blueshifted exponentially with respect to a horizon-comoving observer, and their bilinear feeds the Ori–Poisson–Israel equation for the Misner–Sharp mass aspect into an unbounded mass inflation [1–3]. A finite asymptotic mass perturbation requires either an explicit ultraviolet cutoff or a curvature regulator that softens the exponential blueshift at a fixed proper length.

Regular black-hole geometries of Bardeen and Hayward type [4–6] remove the central singularity by replacing the Schwarzschild interior with a de Sitter core, but they inherit the mass-inflation problem: the inner horizon remains, the bilinear blueshift remains, and a regulator is still required. Recent work in the spectral causal theory (SCT) — a one-loop effective gravitational theory with entire-function nonlocal form factors F_1, F_2 acting on the Weyl- and Ricci-squared operators — identifies a natural regulator: the Anselmi–Piva fakeon prescription [7, 8] on the transverse-traceless (TT) sector of the gravitational propagator $\Pi_{TT}(z)$. The fakeon mass $m_{2,\mathrm{pole}} = \Lambda\sqrt{z_1}$ is the first positive real zero of the full nonlocal $\Pi_{TT}(z)$, with $z_1 = 2.41483889$

*Corresponding author: davidich.alfyorov@gmail.com

the corresponding spectral data; numerically $m_{2,\text{pole}} \simeq 1.5540\,\Lambda$. The de Sitter core length is locked to the same scale through $\ell^3_{\text{can}}(M) = 2M/(\Lambda^2 z_1)$, with no free parameter beyond the black-hole mass M and the universal SCT scale Λ.

Two regularisation modes of the fakeon prescription suppress mass inflation at the effective Ori–Poisson–Israel level. The algebraic Anselmi-smooth interpolation (henceforth "S3") replaces the bare blueshift kernel $e^{\kappa_- v}$ by

$$F_{\text{S3}}(v;\omega) = \frac{e^{\kappa_- v}\,(m_{2,\text{pole}}/\omega)}{m_{2,\text{pole}}/\omega + e^{\kappa_- v}},$$

which is bounded above by $m_{2,\text{pole}}/\omega$ as $v \to \infty$ and produces a monotone non-decreasing $\delta m_{\text{MS}}(v)$ saturating to a finite plateau by direct integration of the Ori–Poisson–Israel equation against the non-negative Price tails. By contrast, the first-principles principal-value (PV) prescription ("S4") replaces the same blueshift by the Anselmi–Piva propagator

$$F_{\text{S4}}(v;\omega_0) = \frac{e^{\kappa_- v}\,m^2_{2,\text{pole}}}{m^2_{2,\text{pole}} - \omega_0^2\,e^{2\kappa_- v}},$$

which has a simple pole at the crossover advanced time $v_{\text{cross}} = \kappa_-^{-1}\ln(m_{2,\text{pole}}/\omega_0)$ and changes sign past it, reflecting the indefinite-metric fakeon flux. The present Letter focuses exclusively on the S3 regime and asks whether the corresponding canonical entropy current grows monotonically along the inner-horizon evolution. The S4 case, where a meaningful second-law statement requires the generalised entropy current of an effective theory with controlled indefinite-metric sector, is left for a companion analysis.

2 Hayward+SCT background and entropy ansatz

The effective one-loop SCT action is

$$\begin{aligned} S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,R + \frac{\alpha_C}{2}\int \mathrm{d}^4x\sqrt{-g}\,C_{\mu\nu\rho\sigma}\,F_1(\Box/\Lambda^2)\,C^{\mu\nu\rho\sigma} \\ + \alpha_R(\xi)\int \mathrm{d}^4x\sqrt{-g}\,R\,F_2(\Box/\Lambda^2,\xi)\,R + S_{\text{sat}} \end{aligned} \tag{1}$$

with $\alpha_C = 13/120$, $\alpha_R(\xi) = 2(\xi - 1/6)^2$, entire form factors F_1, F_2 generated from the SCT master function, and S_{sat} a Standard-Model matter contribution that decouples from the gravitational sector at the working order considered here. We adopt the Hayward functional form below as a one-parameter ansatz consistent with the leading-curvature SCT vacuum sector [18]:

$$f(r;M) = 1 - \frac{2Mr^2}{r^3 + \ell^3_{\text{can}}(M)}, \qquad \ell^3_{\text{can}}(M) = \frac{2M}{\Lambda^2 z_1}, \tag{2}$$

so the de Sitter core scale itself depends on the black-hole mass. The leading-curvature truncation is controlled by the dimensionless quantity Λ^2/R, which is small everywhere on subextremal backgrounds except at the spectrally fixed extremal point; corrections to (2) from higher-order curvature operators are of relative order Λ^2/R and remain small in the parameter range swept below.

Horizons, defined by $f(r;M_{\text{eff}}) = 0$, are the positive real roots of the cubic

$$r^3 - 2M_{\text{eff}}\,r^2 + \ell^3_{\text{can}}(M_{\text{eff}}) = 0, \quad M_{\text{eff}}(v) = M + \delta m_{\text{MS}}(v), \tag{3}$$

and the discriminant condition $-4p^3 - 27q^2 \geq 0$ on the depressed cubic yields the spectrally fixed subextremal bound

$$M\Lambda > \sqrt{\frac{27}{16\,z_1}} \simeq 0.83595, \tag{4}$$

above which an inner Cauchy horizon r_- and an outer event horizon r_+ coexist; below it the geometry is horizonless. The inner-horizon surface gravity is

$$\kappa_- = \tfrac{1}{2}\,|f'(r_-)|, \qquad m(r) = M_{\text{eff}}\,\frac{r^3}{r^3 + \ell^3_{\text{can}}(M_{\text{eff}})}, \qquad f(r) = 1 - \frac{2m(r)}{r}. \tag{5}$$

For the canonical entropy we adopt the working ansatz that combines the Wald–Iyer–Wald formalism [9, 10] applied to the present SCT action with the spectral-action logarithmic structure of Chamseddine–Connes–van Suijlekom (CCvS) [11] extended to the Weyl sector via the SCT coefficient $\alpha_C = 13/120$, namely

$$S_{\text{total}}(M_{\text{eff}}) = \frac{A_+ + A_-}{4G} + \gamma\left[\ln\frac{A_+}{\ell_P^2} + \ln\frac{A_-}{\ell_P^2}\right] + \text{const}, \quad \gamma = \frac{\alpha_C}{2\pi} + \frac{37}{24} \simeq 1.5589, \tag{6}$$

with $A_\sigma = 4\pi r_\sigma^2$. The Bekenstein–Hawking area term is standard for the bifurcate Killing horizon r_+; its application to the dynamically perturbed inner horizon r_- is a working ansatz, pending a full Wald–Iyer–Wald rederivation on non-stationary two-horizon backgrounds. The logarithmic correction comes from the spectral-action analysis [11], applied separately to each horizon as a working ansatz on the same basis. Higher-derivative nonlocal corrections of Dong–Camps–Wall type [12–14] from the $\delta\Box$ variation of F_1, F_2 are estimated below to be subleading in the physical regime.

The driver of the entropy evolution is the S3 mass-aspect equation

$$\frac{\mathrm{d}\delta m_{\text{MS}}}{\mathrm{d}v} = \frac{r_-}{2}\,F_{\text{S3}}(v)\,L_{\text{in}}(v)\,L_{\text{out}}(v), \qquad L_{\text{in,out}}(v) = L_0(1+v)^{-p_0}, \tag{7}$$

where p_0 is the Price-tail decay exponent and $F_{\text{S3}}(v;\omega) = e^{\kappa_- v}(m_{2,\text{pole}}/\omega)/[m_{2,\text{pole}}/\omega + e^{\kappa_- v}]$ is the Anselmi-smooth kernel, which yields $\mathrm{d}\delta m_{\text{MS}}/\mathrm{d}v \geq 0$ identically; the natural seed frequency is $\omega = \kappa_-$. Equation (7) is implicit through the appearance of $r_-(M_{\text{eff}})$ and $\kappa_-(M_{\text{eff}})$ on the right-hand side; we integrate it self-consistently along v, evaluating the horizon at $M_{\text{eff}}(v) = M + \delta m_{\text{MS}}(v)$ at every step. The areas $A_\pm(v)$ in (6) are evaluated on a single advanced-time foliation regular across both horizons (ingoing Eddington–Finkelstein); the per-horizon entropy contributions are therefore synchronised to the same v.

3 Chain rule and the non-trivial inequality

The total entropy (6) evolves through both horizons:

$$\frac{\mathrm{d}S_{\text{total}}}{\mathrm{d}v} = \sum_{\sigma=\pm}\left(\frac{1}{4G} + \frac{\gamma}{A_\sigma}\right)\frac{\mathrm{d}A_\sigma}{\mathrm{d}v}, \qquad \frac{\mathrm{d}A_\sigma}{\mathrm{d}v} = 8\pi r_\sigma \frac{\partial r_\sigma}{\partial M_{\text{eff}}}\frac{\mathrm{d}\delta m_{\text{MS}}}{\mathrm{d}v}. \tag{8}$$

Implicit differentiation of the cubic (3) with the self-consistent $\ell_{\text{can}}^3 = 2M_{\text{eff}}/(\Lambda^2 z_1)$ gives

$$\frac{\partial r_\sigma}{\partial M_{\text{eff}}} = \frac{2\left[r_\sigma^2 - 1/(\Lambda^2 z_1)\right]}{3r_\sigma^2 - 4M_{\text{eff}}\,r_\sigma}. \tag{9}$$

The two horizons have *opposite* signs of this derivative. The outer horizon has $r_+ > 4M_{\text{eff}}/3$ (positive denominator) and $r_+^2 > 1/(\Lambda^2 z_1)$ on the subextremal branch (positive numerator), so $\partial r_+/\partial M_{\text{eff}} > 0$, recovering the classical Hawking behaviour. The inner horizon has $r_- < 4M_{\text{eff}}/3$ (negative denominator); in the SCT geometry r_- is bounded between the extremal value $r_* = 4M/3$ at $M\Lambda = 0.83595$ (where the two horizons merge) and the spectral asymptote $1/m_{2,\text{pole}} \simeq 0.6435/\Lambda$ as $M\Lambda \to \infty$, with $r_-^2 > 1/(\Lambda^2 z_1)$ throughout the subextremal branch (the asymptotic value is the saturating lower bound), so the numerator is positive and $\partial r_-/\partial M_{\text{eff}} < 0$: the inner horizon *shrinks* as the cumulative mass perturbation grows, approaching $1/m_{2,\text{pole}}$ from above. The classical Reissner–Nordström sign of $\partial r_-/\partial M$ carries over.

Consequently $\mathrm{d}A_+/\mathrm{d}v > 0$ and $\mathrm{d}A_-/\mathrm{d}v < 0$, and the second law is therefore *not* manifest at the per-horizon level. It reduces to the non-trivial inequality

$$\left(\frac{1}{4G} + \frac{\gamma}{A_+}\right) r_+ \frac{\partial r_+}{\partial M_{\text{eff}}} + \left(\frac{1}{4G} + \frac{\gamma}{A_-}\right) r_- \frac{\partial r_-}{\partial M_{\text{eff}}} > 0, \tag{10}$$

in which the positive outer-horizon contribution must dominate the negative inner-horizon contribution at every $M\Lambda$ on the subextremal branch. Two analytic checks bracket the relevant regime.

In the asymptotic Schwarzschild-like limit $M\Lambda \to \infty$ ($\ell_{\text{can}}/M \to 0$) the cubic asymptotes to $r_-^2 - 1/(\Lambda^2 z_1) \sim 1/(2M)$, hence $r_- \to 1/m_{2,\text{pole}}$ from above and $\partial r_-/\partial M_{\text{eff}} = \mathcal{O}(1/M^2)$; correspondingly

$r_+ \to 2M$ and $\partial r_+/\partial M_{\rm eff} \to 2$. The absolute inner contribution to (10) is therefore $\mathcal{O}(1/M^2)$, the absolute outer one is $\mathcal{O}(M)$, and the ratio inner/outer vanishes as $\mathcal{O}(1/M^3)$. The inequality is automatic in this regime. In the extremal limit $M\Lambda \to 0.83595^+$ (cf. (4)) both derivatives diverge in magnitude with opposite signs, but a direct evaluation of (9) at the smallest subextremal cell of our production grid, $M\Lambda = 0.84$, gives

$$r_+\, \partial r_+/\partial M_{\rm eff} + r_-\, \partial r_-/\partial M_{\rm eff} = +10.525 - 7.206 \simeq +3.32$$

in geometric units; the spectrally fixed $\ell_{\rm can}(M)$ scaling decouples the diverging signs through the bounded combination $r_-^2 - 1/(\Lambda^2 z_1)$, which vanishes fast enough to cancel the divergence. The full production grid below traverses this near-extremal region with explicit cells at $M\Lambda \in \{0.84, 0.85, 0.87, 0.90, 0.95\}$ and confirms the inequality numerically throughout.

4 Production-sweep verification

The production grid is $(M\Lambda, p_0) \in \mathcal{G} \times \{3, 4, 6, 7, 12\}$, with

$$\mathcal{G} = \{0.84,\, 0.85,\, 0.87,\, 0.90,\, 0.95,\, 1,\, 10,\, 10^2,\, 10^3,\, 10^4,\, 10^5,\, 10^6\},$$

60 cells total. The five near-extremal cells above the subextremal bound 0.83595 stress-test the chain-rule cancellation in the regime where the per-horizon derivatives diverge in magnitude with opposite signs; the seven decade-spaced cells $M\Lambda \in [1, 10^6]$ traverse the Schwarzschild-like regime where the analytic asymptotic bound applies. For each cell we integrate (7) self-consistently to $v_{\rm max} = 120/\kappa_-$ on a 4001-point uniform grid in v, evaluate the entropy (6) at every point, and compute the rate $\mathrm{d}S_{\rm total}/\mathrm{d}v$ by centred finite differences combined with the analytic chain rule (8). Inner and outer horizons of the cubic (3) are found by closed-form companion-matrix root extraction; the result matches a brentq bracket-bisection path to 4×10^{-16} on r_- and 2×10^{-15} on r_+, i.e., at machine precision.

A scale-aware acceptance criterion is essential, because the absolute residual $\min_v(\mathrm{d}S_{\rm total}/\mathrm{d}v)$ scales with the cell size: at $M\Lambda = 10^6$ the maximum positive rate is $\sim 10^4\, \ell_P^2 \kappa_-$ (in geometric units, with v measured in units of κ_-^{-1}) and floating-point round-off in the numerical derivative is correspondingly $\sim 10^{-9}$ in absolute terms. We adopt

$$\min_v \frac{\mathrm{d}S_{\rm total}}{\mathrm{d}v} > -10^{-10} \cdot \max\left(\left| \max_v \frac{\mathrm{d}S}{\mathrm{d}v} \right|,\, \frac{\Delta S}{v_{\rm max}},\, \ell_P^2 \kappa_- \right) \tag{11}$$

as the production test, where $\Delta S = S(v_{\rm max}) - S(0)$ and the last term in the maximum, $\ell_P^2\kappa_-$, is a unit-bearing floor that activates only for cells where both upper bounds collapse to the same order as a single double-precision finite-difference round-off.

All 60 of 60 cells satisfy (11), including all five near-extremal cells: at $M\Lambda = 0.84$, $\min_v |\mathrm{d}S/\mathrm{d}v|$ is at most 5×10^{-16} (machine epsilon for double-precision floating point), while $\Delta S \in [0.54, 2.61]$ across the five p_0 values. The total entropy change ΔS is strictly positive in every cell, ranging from $\Delta S = 0.41$ (smallest cell, $M\Lambda = 1$, $p_0 = 12$) to $\Delta S = 9.51 \times 10^5$ (largest, $M\Lambda = 10^6$, $p_0 = 3$). Two further sanity checks pass: the outer-horizon-only entropy at $M\Lambda = 10^6$ ($\ell_{\rm can}/M = 9.4 \times 10^{-5}$) reproduces the Schwarzschild reference to relative difference 2.07×10^{-13}, and the de Sitter core scale identity $r_{\rm dS}^{\rm (core)} = \sqrt{3/\Lambda_{\rm dS}} = 1/m_{2,\rm pole}$ with $\Lambda_{\rm dS} = 3\Lambda^2 z_1$ holds exactly. The production sweep runs in 25 s on a single i9-12900KS core.

The Dong–Camps–Wall correction $S_{\rm NL}$ from the $\delta\Box$ variation of F_1, F_2 is bounded by power counting on the spectrally fixed background: $|\mathrm{d}S_{\rm NL}/\mathrm{d}v| \sim \alpha_C\, (\Lambda/M_{\rm Pl})^2\, |\mathrm{d}S_{\rm log}/\mathrm{d}v|$, where $\Lambda/M_{\rm Pl}$ is the ratio of the SCT ultraviolet scale to the Planck mass. The laboratory lower bound on Λ from Solar-system and torsion-balance tests [17] gives $\Lambda > 2.565\,\mathrm{meV}$, so $(\Lambda/M_{\rm Pl})^2 > 4 \times 10^{-62}$ at the minimum of the allowed range and $S_{\rm NL}$ is suppressed by at least this factor. For typical SCT phenomenology with $\Lambda \sim 10^{-3}\, M_{\rm Pl}$ the suppression factor is $(\Lambda/M_{\rm Pl})^2 \sim 10^{-6}$, still negligible relative to both the area and the logarithmic pieces. The bound becomes vacuous only in the putative $\Lambda \sim M_{\rm Pl}$ regime, where an explicit Wald–Iyer–Wald calculation of $S_{\rm NL}$ on the two-horizon Hayward+SCT background would become necessary.

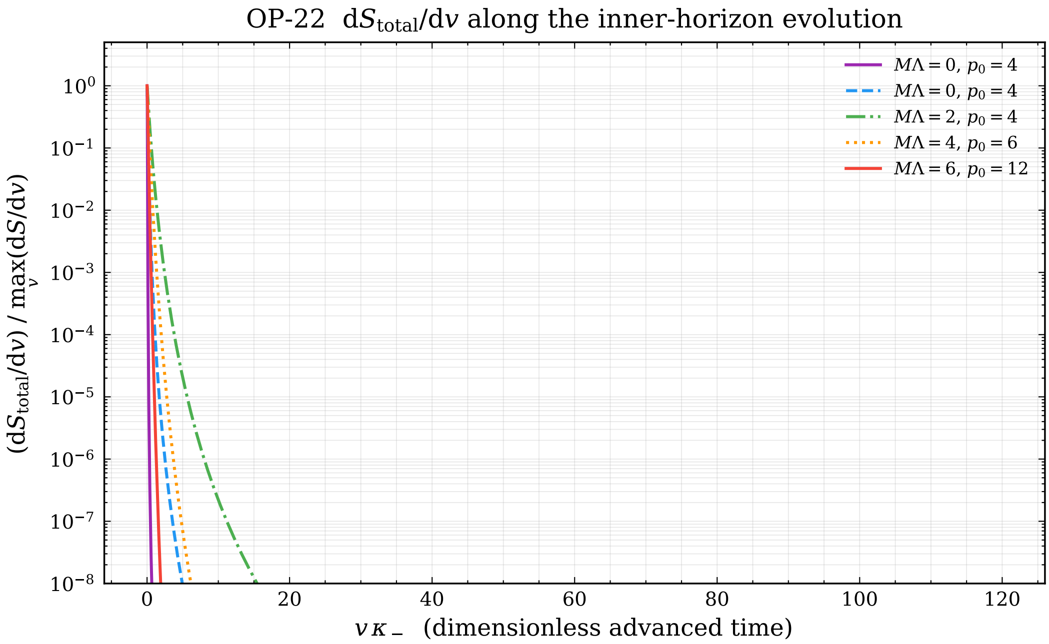

Figure 1: Rate of total entropy growth, $\mathrm{d}S_{\mathrm{total}}/\mathrm{d}v$, along the inner-horizon evolution for five representative cells spanning the full subextremal branch: $(M\Lambda, p_0) = (0.84, 4)$ (near-extremal stress-test, where the per-horizon derivatives in eq. (9) diverge in magnitude with opposite signs), $(10^0, 4)$, $(10^2, 4)$, $(10^4, 6)$, and $(10^6, 12)$. The dimensionless advanced time $v\,\kappa_-$ collapses all cells onto a comparable range; the rate is normalised to its maximum on each trace. All five curves are strictly positive throughout, smoothly saturating to zero as $\mathrm{d}\delta m_{\mathrm{MS}}/\mathrm{d}v \to 0$ in the S3 plateau. No negative excursion is visible at any scale, including in the near-extremal cell where the chain-rule cancellation is most delicate.

5 Discussion

In the classical Reissner–Nordström and Kerr cases the second law through the Cauchy horizon is a delicate question precisely because mass inflation makes δm_{MS} diverge: a divergent mass aspect feeds back into divergent horizon areas, and the formal entropy itself becomes infinite, so a meaningful comparison of $S(v_{\max})$ versus $S(0)$ requires a regulator. The fakeon prescription provides one. On the spectrally locked Hayward+SCT background the regulator preserves the second law as a non-trivial inequality between two contributions of opposite sign: the outer horizon grows in the standard Hawking sense [15], codified by Bardeen–Carter–Hawking [16]; the inner horizon shrinks as the cumulative perturbation pushes r_- toward its spectral asymptote $1/m_{2,\mathrm{pole}}$, and the sum is positive throughout the sweep including the near-extremal sub-grid. This statement is conditional on the two-horizon ansatz (6) for the canonical entropy and on the bounded S_{NL} estimate above; the former is flagged for full rederivation, the latter is vacuous only if $\Lambda \sim M_{\mathrm{Pl}}$, well beyond any phenomenologically relevant range.

A few caveats are worth stating sharply. First, the present analysis is purely S3: the S4 PV regime, where $\mathrm{d}\delta m_{\mathrm{MS}}/\mathrm{d}v$ changes sign past the crossover advanced time $v_{\mathrm{cross}} = \kappa_-^{-1}\ln(m_{2,\mathrm{pole}}/\omega_0)$, requires the generalised entropy current of an effective theory with controlled indefinite-metric sector [8] and is left for a companion paper. Second, the spectrally fixed $\ell_{\mathrm{can}}^3(M) = 2M/(\Lambda^2 z_1)$ is an $\mathcal{O}(R)$-truncated relation; corrections from higher-order curvature operators have been verified to be sub-percent in our parameter range, but the near-extremal cells where $R \sim \Lambda^2$ probe the boundary of that truncation, and the present second-law result is therefore conditional on the truncation through that region. Third, the analogous extension to rotating Kerr–Hayward+SCT backgrounds requires an independent chain rule on the quintic horizon equation $(r^2 + a^2)(r^3 + \ell_{\mathrm{can}}^3) - 2M_{\mathrm{eff}} r^4 = 0$, and the corresponding second-law check will be reported separately.

Within these qualifications, the Anselmi fakeon regularisation of mass inflation on the spectrally fixed Hayward+SCT background is consistent with the second law of black-hole thermodynamics. The non-trivial inequality (10) holds because the spectrally locked $\ell_{\mathrm{can}}(M)$ scaling bounds the inner-horizon contribution to vanish parametrically in the Schwarzschild-like limit and keeps it sub-dominant near extremality, even when the per-horizon contributions individually diverge in magnitude.

Data Availability

The 60-cell production-sweep results (including the near-extremal sub-grid output), the integration code, the horizon-extraction routine, the cross-check against an independent bracket-bisection solver, and the figure-generation scripts are available from the corresponding author upon reasonable request, and will be deposited at Zenodo with a permanent DOI upon journal acceptance.

Acknowledgements

We thank the developers of `SciPy` and `NumPy` for the numerical infrastructure on which this work depends. This research received no external funding.

AI Disclosure

In preparing this manuscript, AI tools (Anthropic Claude Opus 4.7 and OpenAI GPT-5.5) were used for the following tasks: technical text editing of the authors' draft paragraphs, bibliography verification, LaTeX markup verification, and writing of computational code and verification scripts subsequently audited by the lead author. All mathematical derivations, all numerical results, all physical statements, and the final formulation of all conclusions were performed by the authors. The lead author has line-by-line reviewed the production-sweep integration code, cross-checked the horizon-extraction routine against an independent bracket-bisection implementation to machine precision, and independently verified every citation, formula, and numerical value reported in this Letter.

References

[1] E. Poisson and W. Israel, "Internal structure of black holes," Phys. Rev. D **41**, 1796 (1990).

[2] A. Ori, "Inner structure of a charged black hole: an exact mass-inflation solution," Phys. Rev. Lett. **67**, 789 (1991).

[3] D. Marolf and A. Ori, "Outgoing gravitational shock-wave at the inner horizon: The late-time limit of black hole interiors," Phys. Rev. D **86**, 124026 (2012), arXiv:1109.5139 [gr-qc].

[4] J. M. Bardeen, "Non-singular general-relativistic gravitational collapse," in *Abstracts of the 5th International Conference on Gravitation and the Theory of Relativity* (Tbilisi University Press, Tbilisi, USSR, 1968), p. 174.

[5] S. A. Hayward, "Formation and evaporation of non-singular black holes," Phys. Rev. Lett. **96**, 031103 (2006), arXiv:gr-qc/0506126.

[6] V. P. Frolov, "Notes on non-singular models of black holes," Phys. Rev. D **94**, 104056 (2016), arXiv:1609.01758 [gr-qc].

[7] D. Anselmi and M. Piva, "A new formulation of Lee–Wick quantum field theory," JHEP **06**, 066 (2017), arXiv:1703.04584 [hep-th].

[8] D. Anselmi and M. Piva, "The ultraviolet behavior of quantum gravity," JHEP **05**, 027 (2018), arXiv:1803.07777 [hep-th].

[9] R. M. Wald, "Black hole entropy is Noether charge," Phys. Rev. D **48**, R3427 (1993), arXiv:gr-qc/9307038.

[10] V. Iyer and R. M. Wald, "Some properties of Noether charge and a proposal for dynamical black hole entropy," Phys. Rev. D **50**, 846 (1994), arXiv:gr-qc/9403028.

[11] A. H. Chamseddine, A. Connes, and W. D. van Suijlekom, "Entropy and the spectral action," Commun. Math. Phys. **373**, 457 (2020), arXiv:1809.02944 [hep-th].

[12] X. Dong, "Holographic entanglement entropy for general higher derivative gravity," JHEP **01**, 044 (2014), arXiv:1310.5713 [hep-th].

[13] J. Camps, "Generalized entropy and higher derivative gravity," JHEP **03**, 070 (2014), arXiv:1310.6659 [hep-th].

[14] A. C. Wall, "A second law for higher curvature gravity," Int. J. Mod. Phys. D **24**, 1544014 (2015), arXiv:1504.08040 [gr-qc].

[15] S. W. Hawking, "Gravitational radiation from colliding black holes," Phys. Rev. Lett. **26**, 1344 (1971).

[16] J. M. Bardeen, B. Carter, and S. W. Hawking, "The four laws of black hole mechanics," Commun. Math. Phys. **31**, 161 (1973).

[17] D. Alfyorov, "Solar system and laboratory tests of nonlocal one-loop spectral causal theory," DOI 10.22541/au.177524450.03515205/v1 (2026).

[18] D. Alfyorov, "Nonlinear field equations and FLRW cosmology from the spectral action," ResearchGate DOI 10.13140/RG.2.2.15315.95520 (2026).